

## **Evaluation of Granular-Fill Damping In a Shock-Loaded Box Beam**

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The experimental evaluation of damping performance in a granular-material-filled box beam is presented. The study determined that granular-fill provided significant damping for a high impact excited free-free box beam. A large modal loss factor, exceeding 0.15, was obtained for the few lowest bending modes of the beam. The damping was effective over the frequency range 40 - 300 Hz. A variety of data analyses and modeling methods were used to assess the damping effects from acceleration data. These methods included a novel "complex wave number method" in which the damping values were determined for different wave types over discrete, but regularly-spaced, frequency values. These results were compared with conventional methods such as a modal analysis approach.

### **INTRODUCTION**

The science of granular materials has a long history. Despite its seeming simplicity, granular materials behave differently from the other familiar forms of matter: solids, liquids or gases. One of the unique properties of a granular material, such as a low-density polyethylene bead, is that interactions between bead particles are dissipative due to static friction and inelastic collisions [1].

While a variety of approaches exist for modifying the damping characteristics of structures constructed from an individual beam, the filling of box beams with granular material has been shown to produce the most significant increase in damping [2]. With proper selection of fill material, such as a bead, significant damping can be achieved with only a modest weight penalty. However, it is clear from existing literature that the mechanism responsible for the damping effectiveness of the granular material differs from that of other more conventional damping treatments.

The work presented in this paper was motivated by the need to determine the dynamic property and damping effectiveness in shock-loaded truss structures composed of many granular material filled box-beams. The damping loss factor<sup>1</sup> associated with each beam may depend on the properties of the fill material, the beam geometry, the load, and their dynamic interactions. This is, in general, difficult to model by analytical methods or numerical simulation. The focus of this work is to determine damping loss factor for a granular material-filled individual beam member when subjected to large impact force excitations. This is achieved by performing experiments and

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<sup>1</sup> Damping loss factor  $\eta$  is equal to twice the critical damping ratio at structural resonance.

using the experimental data to assess damping in the frequency range of interest. The knowledge gained from this study should improve the fundamental understanding of structural behavior of a granular-damping-treated beam and provide the basis for further analysis and modeling of a built-up shock loaded truss structure.

## APPROACH

A simple experiment was performed to quantitatively assess damping characteristics using a box-beam of typical size in naval applications with two types of granular-materials as damping treatment. The test beam was subjected to hammer impacts at its end. This resulted in average overall peak acceleration levels of the beam in the range of 5 - 50g's. The measured acceleration data were used to determine the damping performance. This included: (1) the determination of shock benefits provided by the damping material and the quantification of those benefits; (2) the determination of the frequency range over which the damping mechanisms is effective (emphasis on frequencies under 300 Hz); and (3) the determination of the relationship between the dynamic g-levels and the measured damping provided by the materials as well as the examination of the energy absorption mechanism for the bead-filled test beam.

Initially, a traditional approach to the analysis of such experimental data sets was used. This may be referred to as a modal approach, where conventional modal parameter estimation and extraction algorithms were employed. The main portions of data analysis were performed using the Complex Exponential Method and Ibrahim Time Domain Method [3]. These algorithms work on measured data in a time domain format, rather than the frequency domain, and extract the damping factor at specific resonant frequencies corresponding to the test specimen. The results of the modal approach can be found in the data analysis section.

A wave model has been developed to analyze the experimental beam data. In this approach, the response along the entire beam at a particular frequency is represented by a fixed number of damped wave types [4]. A brief description of the method is as follows. Based on Euler-Bernoulli theory, a differential equation of a free vibrating beam with no external loads can be written as

$$\rho A \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left[ EI(x) \frac{\partial^2 w}{\partial x^2} \right] = 0 \quad (1)$$

where  $w$  is the transverse displacement of any point on the axis of the beam,  $EI$  is the flexural rigidity of the beam in the plane of vibrations, and  $\rho A$  is the mass per unit length of the beam. With a periodic forcing applied at the end of beam, the solution of the above equation is a sum of two forward and two backward propagating waves,

$$w(x, t) = \Re \left\{ \sum_{n=1}^2 F_n e^{i(k_n x - \omega t)} + B_n e^{i[k_n(L-x) - \omega t]} \right\} \quad (2)$$

where  $\Re$  refers to the real part. The index  $n=1,2$  refers to flexural and evanescent waves, respectively. The complex wave numbers are

$$k_1 = \sqrt[1/4]{\frac{\rho A \omega^2}{EI}} \quad (3)$$

$$k_2 = i \sqrt[1/4]{\frac{\rho A \omega^2}{EI}} \quad (4)$$

These wave types can be identified from measured acceleration data shown in the data analysis section. Damping loss factor  $\eta$  is introduced through a complex modulus of elasticity  $E(1-i\eta)$ .

In addition to the flexural and evanescent waves described above, a box beam also supports plate waves. Each wall of a box beam flexes in and out as plate waves travel down the length of the beam. This type of wave motion can be characterized using Kirchhoff's plate theory. The frequencies of the plate waves for the test beam are above 500 Hz. Since we are focusing on frequencies below 300 Hz, the plate waves were not included in the wave representation of the damping model in this paper.

The key ideas of identifying damping factors by wave model includes several steps. The first step is to convert transient response data in the time domain to independent forced response data at discrete frequency values in the frequency domain using Fast Fourier Transforms (FFT). The next step is to decompose each forced response as a sum of waves and fit the wave field to the measured data. Mean square errors between wave field and data are then minimized by adjusting complex wave numbers. Finally, the damping loss factor  $\eta$  is calculated from the above equations using an error-minimized complex wave number  $k$  at each FFT frequency. A complete description of the method can be found in [4].

## EXPERIMENTAL ARRANGEMENT

### Description of the Box-Beam

A rectangular cross section 10" X 6" X 3/8" box-beam, 16 ft in length, was used in the impact test. The beam weight was estimated to be approximately 603 lbs by calculation. The natural frequencies (in Hz) of the five lowest transverse vibrational (bending) modes for a free-free boundary condition beam are listed in the table below

y X x = 10 X 6	1st mode	2nd mode	3rd mode	4th mode	5th mode
x - x axis	71	194	381	630	941
y - y axis	47	131	256	423	632

The lowest longitudinal and torsional modes of the beam are above 500 Hz, more than 200 Hz above the maximum frequency of interest. Therefore, these modes were ignored in the experimental design and analysis.

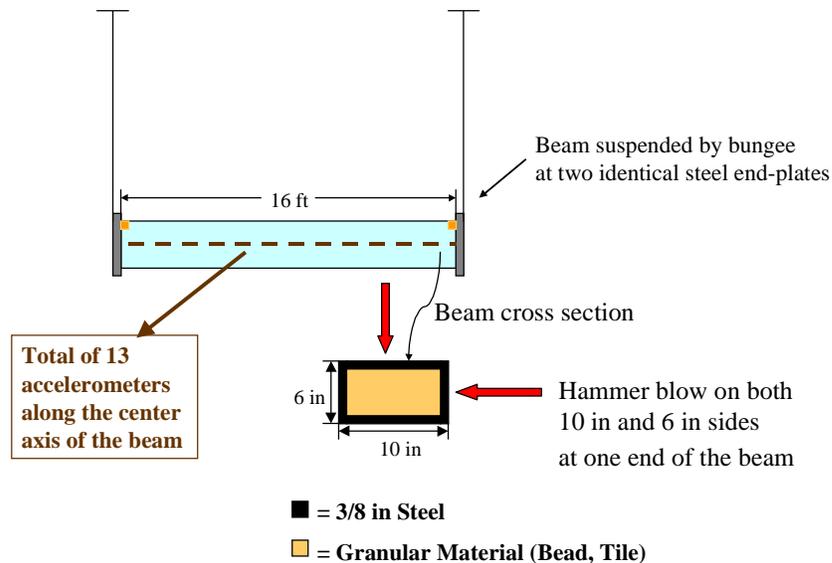
Steel plate endcap assemblies, with dimensions of approximately 12" X 8" X 3/4", were bolted to both ends of the beam for convenient installation of fill materials. The two end plates had identical geometrical configurations. The total weight of the beam with endcaps measured 610 lbs.

### Fill Material

Shredded Navy tiles (type II, class II) and low-density polyethylene beads (Chevron LDPE 1117B) were used as granular fill material in these tests. The measured bulk density of LDPE beads is 36.8 lbs/ft<sup>3</sup>. The volume available to the fill material is 5.40 ft<sup>3</sup>. The total weight of the beads needed to fill the beam is approximately 200 lbs. The density of the shredded Navy tile is only slightly larger than that of the beads.

## Test Configuration

The test was conducted by suspending the beam horizontally with an elastic cord, approximating free-free boundary conditions. The low-frequency suspension cord was used to ensure that the dynamics of the suspension system would not affect the modal response of the beam. The frequency of this suspension system was estimated to be less than 5 Hz. The suspended beam motion is similar to that of a pendulum. The few possible "rigid body motions" of the pendulum have frequencies estimated to be less than a few Hz. The two transverse directions of the beam were excited separately to obtain the frequencies corresponding to those tabulated in the above table. The excitation forces were generated by an instrumented impact hammer striking at one end of the beam on the side opposite to the accelerometers. The test configuration is shown in Fig. 1.



**Figure 1.** Beam Impact test set-up

Two different hammer tip materials were used during the test. To concentrate the excitations in the lower frequency range, the softer hammer tip was used. The duration of the impulse generated from this material is typically between 15 to 20 milliseconds. These results can be compared to those generated by the harder material tip, which resulted in force pulse duration of approximately 5 to 10 milliseconds. The harder material tip provided energy over a broader frequency range.

A variety of excitation magnitudes were recorded in the test series. It ranged from a few hundred pounds up to 8,000 pounds. The initial peak response acceleration levels were generated from a few g's to above 50 g's. The measured data can be utilized to study the amplitude dependence of the damping.

The test series was conducted on an empty beam to provide the undamped case as a baseline. Two types of granular-fill material, each at the 100% and approximately 80% fill levels were used separately to fill the beam. During the fill process, vibration was applied to the beam (e.g. by hammer tapping) prior to data collection to ensure the granular material was well settled in the beam.

## Instrumentation and Data Acquisition

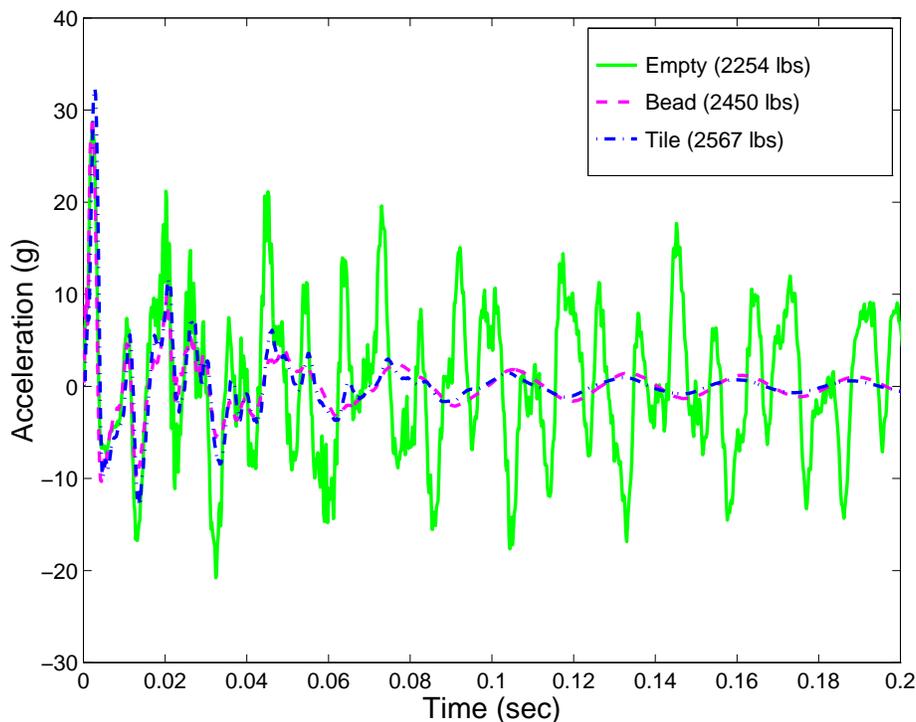
Accelerations at 26 different response locations were measured in transverse directions along two sides of the beam, 13 locations on each side. Accelerometers were spaced approximately 16 inches apart which provided approximately 6 points per wave for the 4th bending mode (400-600 Hz). This arrangement can provide spatial damping performance measurements in addition to temporal measurements.

The uniaxial accelerometers used in the test can measure accelerations up to 50g. The excitation forces at the impact location were also recorded. A data record length of 4 seconds at a sampling rate of 5,000 Hz was stored for each measurement. The final digitized data were stored in ASCII format for analysis and damping evaluation.

## DATA ANALYSIS

### Beam Time Series Data

The test series was conducted on an empty beam to provide the undamped case as a baseline. Typical accelerometer data comparing empty beam and 100% bead and tile material filled beam is shown in Fig 2.



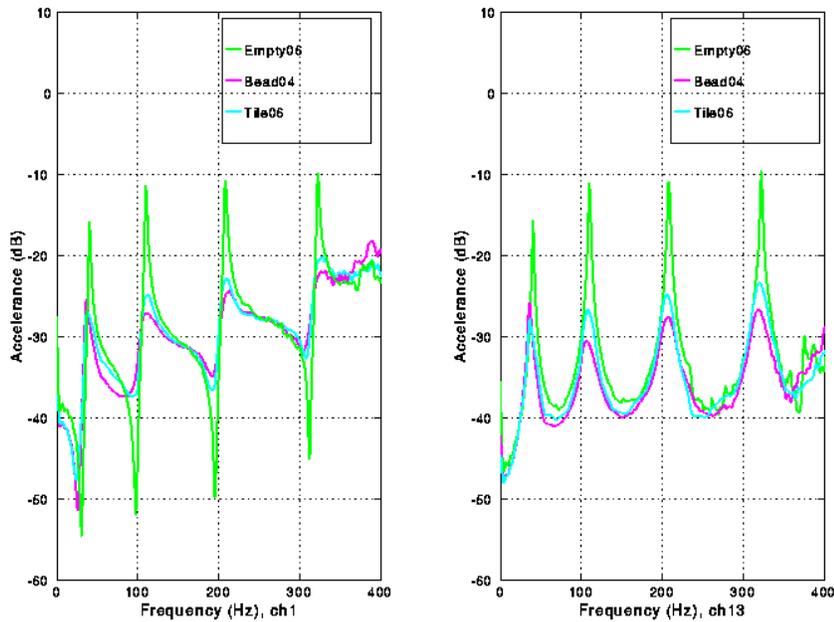
**Figure 2.** Empty beam is compared with 100% bead- and tile-filled beam. Impact force is approximately 2,400 lbs with pulse duration 10 msec on 10 inch side of the beam. The measured accelerations shown are taken from the nearest channel to the impact point (at  $x=0$ ).

## Modal Characterization of Beam Data

Frequency response functions were calculated using the measured data from the example shown in Fig. 2. These functions were computed using a standard force normalization procedure, namely the measured accelerations were normalized by impact forces with exponential windowing [3]. The frequencies of the lowest three modes for the empty beam were 40.4, 110.2, and 208.1 Hz for excitations on the 10" side (See Fig. 3) and 60.3, 163.2, and 310.5 Hz for excitations on the 6" side. The measured frequencies from experimental data were slightly lower compared with the analytical calculations tabulated in previous section. The analytical calculations were done for a free-free beam without end plate.

To check for nonlinearity of the test beam, acceleration measurements were repeated many times using different levels of excitation each time. The amplitude of the frequency response function for the empty beam is independent of excitation forces that ranged from a few hundred pounds to six thousand pounds. The amplitude of the frequency response function for the filled beam (both bead and tile) showed clear variation for the same input force range indicating nonlinearity.

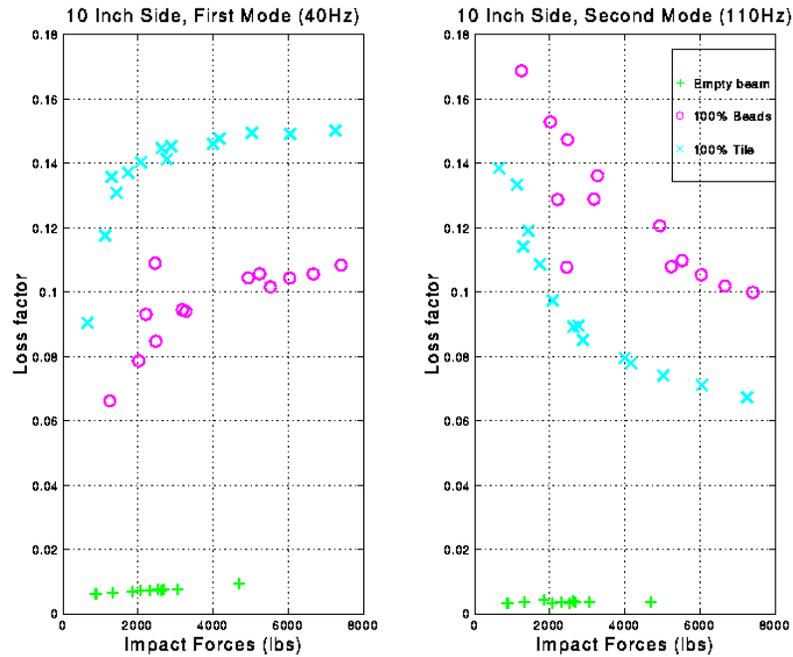
As a means of investigating this nonlinear behavior, a range of impact levels was applied to excite the granular-filled beam in the experiment. Modal estimation and extraction algorithms were then used to determine the damping loss factor for a given impact level. The damping value obtained with this approach, therefore, corresponds to a particular impact level only.



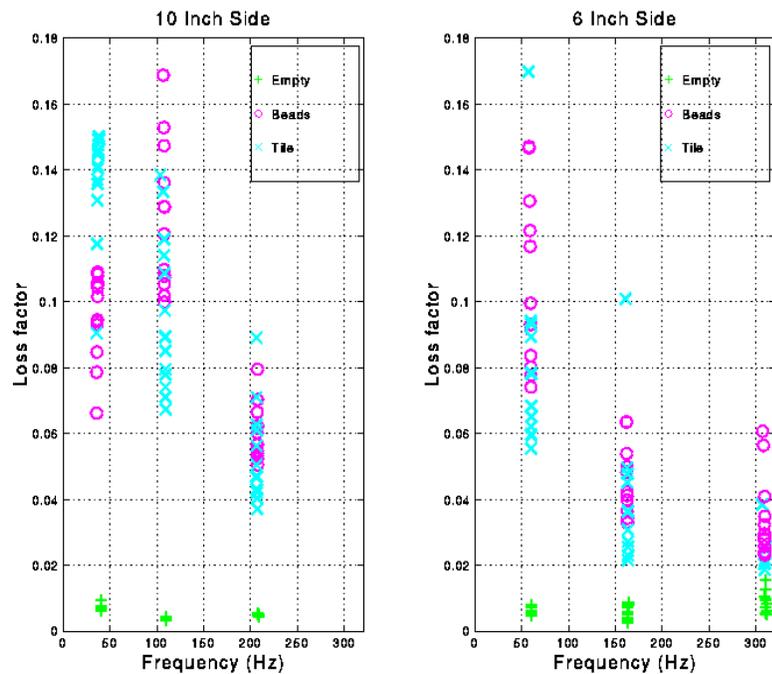
**Figure 3.** Frequency response function calculated using measured acceleration data normalized by force. Input forces are approximately the same (2,400 lbs) for the empty beam, 100% bead- and tile-filled beam. The plot on the left side corresponds to the measurement closest to the data taken from impact point ( $x = 0$ ) and the right side corresponds the data taken from the opposite end of the beam ( $x = L$ ).

A time duration of 500 msec (correspond to 2,500 data points) was used for all damping loss factor calculations, starting at the first zero crossing of the impact point acceleration (taken from the accelerometer located at  $x=0$ ). Data from all 13 accelerometers were used simultaneously. In Fig. 4, the damping loss factor  $\eta$  is displayed for the empty beam, 100% bead-filled beam and 100% tile filled beam for the lowest two modes corresponding to excitations on the 10" side. The left side of the plot shows higher damping for tile-filled beam compared to the bead-filled beam and damping increases for both types of fill-material with impact force amplitude (the first mode

at 40.4 Hz). The right side of the plot shows higher damping for bead-filled beam compared to the tile-filled beam and damping decreases for both types of fill-material with impact force amplitude (the second mode at 110.2 Hz).



**Figure 4.** Modal loss factor versus excitation level on 10" side of the beam for empty beam (plus), 100% bead (circle) - and tile (cross) -filled beam. The plot on the left side is for the first mode (at 40.4 Hz) and on the right side is for the second mode (at 110.2 Hz).



**Figure 5.** Modal loss factor versus modal frequency for the empty beam (plus), 100% bead (circle) - and tile (cross) -filled beam. The plot on the left side is for excitations on 10" side and on the right side is for excitations on 6" side.

These results strongly indicate that granular-material-damping behavior is a function of response amplitude as well as frequency. To examine the damping performance as a function of frequency, modal loss factors for the empty beam are compared to those for the granular-filled-beam for the three lowest modes on both 10" and 6" sides (Fig. 5). The variation of damping values at particular modal frequencies is primarily due to the different levels of excitation (approximately 600 lbs – 7,000 lbs).

The modal characterization of the granular-material-filled beam indicated that damping exhibits complex dependence on both frequency and amplitude. A more fundamental understanding is needed in order to successfully model this complicated damping behavior. However, the large damping loss factor indicates that the granular damping treatment for box beam under impact load is effective.

## Wave Approach

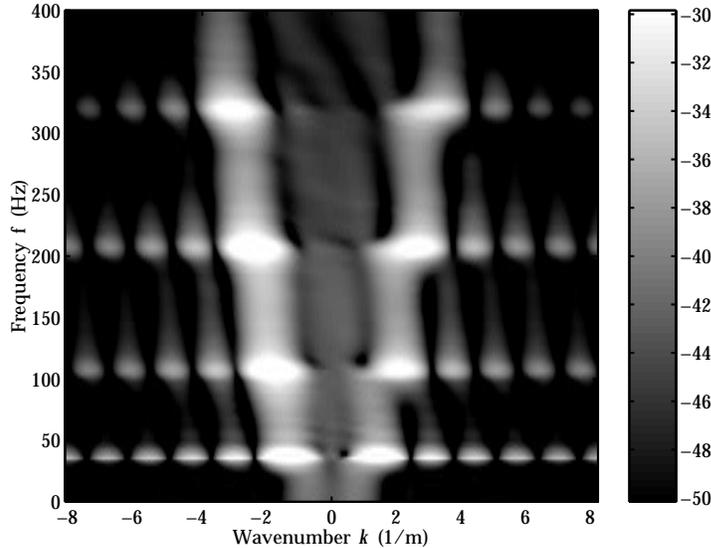
Modeling the frequency-dependent damping behavior from measured data, the wave method was developed to obtain the damping loss factor as a continuous function of frequency. The presence of waves in the measured acceleration data  $a(x, t)$  can be verified by taking a two-dimensional Fourier transform

$$A(\omega, k) = \frac{1}{(2\pi)^2} \int_0^T \int_0^L a(x, t) e^{i(kx + \omega t)} dx dt \quad (5)$$

The transform  $A(\omega, k)$  is maximized whenever the wave number and frequency  $(k, \omega)$  coincide with that of a natural wave,  $(k_n, \omega_n)$  as described by

$$a(x, t) = \Re\{A' e^{i(k_n x - \omega_n t)}\} \quad (6)$$

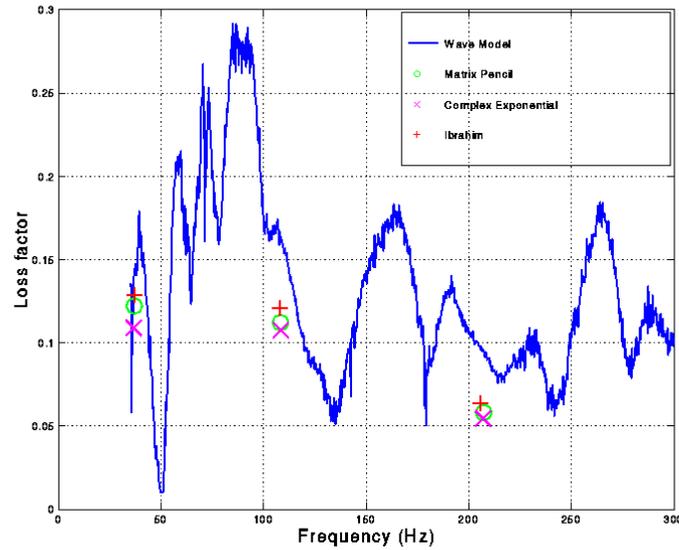
The wave propagation in  $k$ - $\omega$  plane is given in Fig. 6.



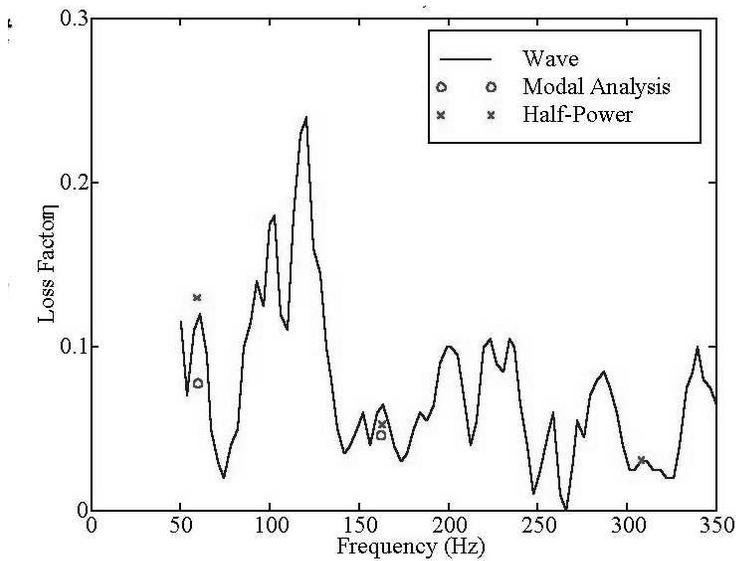
**Figure 6.** Frequency-wave number transform of acceleration data for the bead-filled beam subjected to a typical impact. The excitation is applied on the 10" side of the beam.

The two branches of the parabola in the  $k$ - $\omega$  plane are clearly visible in the frequency range  $0 \leq f \leq 300$  Hz. They are particularly strong at the modal frequencies of the beam. The side lobes to the left and right of this parabola are due to the coarse spacing of the sensors (a total of 13 accelerometers). The left branch of the parabola ( $k < 0$ ) corresponds to the wave leaving the impacted end of the beam and the right branch ( $k > 0$ ) corresponds to the wave reflected from the far end. It can be seen that the magnitude of the reflected wave is reduced.

The damping loss factor  $\eta$  can be calculated by the wave model using the procedure described in [4]. Examples of the loss factor  $\eta$  for the bead-filled beam obtained from the wave method are compared with those from several modal methods at the first three modal frequencies in Fig. 7 (excitations on 10" side) and Fig. 8 (excitations on 6" side).



**Figure 7.** Damping loss factor versus frequency for bead-filled beam determined by wave and modal methods. The excitations are on the 10" side of the beam.



**Figure 8.** Damping loss factor versus frequency for bead-filled beam determined by wave and modal methods. The excitations are on the 6" side of the beam.

Good agreement is observed in Fig. 7 for the first mode and for all modes in Fig. 8, but further investigation is needed to understand the differences for the second and third modes in Fig. 7. Both methods exhibit larger damping for excitations on the 10" side where the beam has lower structural stiffness. A possible explanation is that higher

energy levels can be transferred to the bead-material. Characteristic peaks seen in the wave model suggest that the damping due to granular-fill material is strongly dependent on frequency. The frequency dependent behavior for steady state excitations of a beam has been modeled by others. The average wave model estimate for  $\eta$  exceeds the average modal estimate. It is likely that the difference can be attributed to the difference in the data record length used for the two methods. The reason for this difference in record lengths involves fundamental issues of the two methods. Detailed explanations are given in [4].

## SUMMARY

Two distinct approaches were used in the analysis of shock-loaded beam (empty and granular-filled) experimental data. In the conventional modal approach, values of damping loss factors are obtained only for the natural frequencies of the test structure. With the wave model, the damping factor is extracted at each frequency bin of a temporal Fourier transform of the data. The resulting frequency-dependent damping factor can then be applied to predicting the behavior of built-up structures whose modal frequencies do not coincide with those of the test specimen.

Damping benefits provided by granular-material-filled box beam were evaluated and quantified over the frequency range 40 - 300 Hz. These evaluations were carried out for beam transient response data. A variety of impact forces were applied to the test beam, resulting in amplitude- and frequency-dependent damping. Analysis of the test data indicates that granular material provided significant damping for an impact-excited box-beam. The performance of the Type II tile and LDPE beads were comparable, with the tile exhibiting better damping performance at the first mode in some cases. From analysis of the data, the average damping loss factor ranged from  $0.06 \leq \eta \leq 0.15$  for the filled beam; for the empty beam, damping loss factor ranged from  $0.005 \leq \eta \leq 0.008$ . Higher damping was observed for lower structural stiffness under the same excitation forces. In addition, higher damping occurred for lower initial amplitude levels (except for the first mode at 40 Hz) in the range of approximately 7g to 50g. Finally, damping increased with the fill level, that is, the higher the fill level in the beam the better the damping.

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