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Distinguishability and Identifiability of Contact States

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Abstract **- An important component of machine perception is the estimation of contact states during task execution. This paper addresses two fundamental questions that must be answered when formulating mathematical descriptions of the contact states: Are two contact states distinguishable from each other? Can the unknown or imprecisely known parameters in these descriptions be identified? A technique is presented to answer these questions, which is based on a Taylor series expansion of the contact state's constraint equations. The approach is illustrated through several examples.**

Index Terms **- distinguishability, identifiability, contact state estimation, machine perception.**

I. INTRODUCTION

Enhancing machine perception is an important component of achieving robot autonomy. To this end, the literature contains a growing body of work on contact state estimation, e.g., [6], [14], [9], [16]. This approach to perception is based on the concept that tasks can be decomposed into sequences of contact states. At each step of task execution, motion planning and control involve moving from one contact state to another.

Published applications of contact state estimation include cooperative human and machine perception during teleoperation [7], assembly task monitoring [16], automated virtual model calibration [8], and force control [4]. While the implementations of contact state estimators differ in these works, each method is based on a mathematical description of the contact states.

The description, in the form of constraint equations, can involve the kinematics as well as force-velocity relationships. These equations are parameterized by robot sensor data and by the physical parameters of the objects in contact (e.g., geometry, mass, friction).

A rigorous approach to estimator design involves ensuring that the proposed set of sensors and associated contact constraint models are sufficient (1) to distinguish each contact state from the others in the task, and (2) for each contact state, to identify the objects' physical parameters. In most prior work on contact state estimation, however, the distinguishability and identifiability of candidate constraint models can only be confirmed ex post facto through local numerical tests (e.g., residual analysis) of experimental data. This approach necessitates experimental testing of a set of candidate models, along with their associated sensors, to determine their feasibility for contact state estimation

In contrast, this paper presents an analytical method that can be used to deduce contact state estimation feasibility without the need for experimental implementation. In this approach distinguishability and identifiability are defined as dual problems that can be solved in a systematic manner for any set of sensors and constraint equations. Thus, the method can be used to select sensors and constraints independent of the technique selected to perform the experimental contact state estimation.

The paper is arranged as follows. Section II provides an overview of distinguishability and identifiability in the literature. Section III presents the proposed method. Examples are provided in section IV followed by conclusions in the final section.

II. BACKGROUND

As an example of contact states, two polygonal contact states commonly used in planar peg insertion are considered in Fig. 1. Contact state 1 corresponds to the contact between a vertex of a manipulated polygon and an edge of a fixed polygon (i.e., grey colored block). Contact state 2 describes the contact between an edge of the manipulated polygon and a vertex of a fixed polygon.

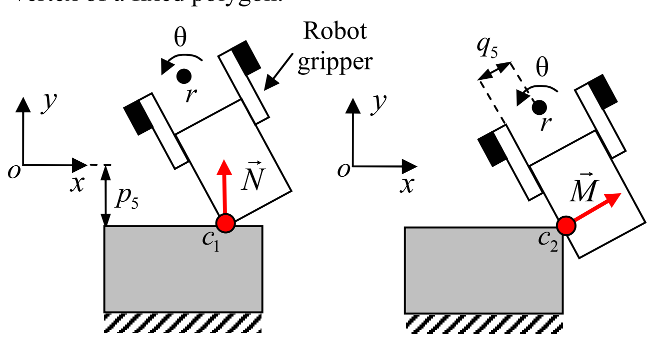

Fig.1 a) Contact state 1 b) Contact state 2

The vectors $\vec{N} = [p_1, p_2]$ and $\vec{M} = [q_1, q_2]$ correspond to the unknown orientations of the contact normals. The unknown locations of the contact points are $\overrightarrow{rc_1} = [p_3, p_4]$ and $\overrightarrow{oc_2} = [q_3, q_4]$, respectively. The parameters p_5 and q_5 represent the positional offset in the direction of the normal for the fixed edge in contact 1 and the manipulated edge in contact 2, respectively. The vector $\overrightarrow{or}$ corresponds to the location and orientation of the gripper with respect to the robot base frame. These examples will be used in the remainder of the paper.

Using techniques described in our previous papers [5], [6], the constraint equations for these two contact states can be written as follows.

Contact State 1

$$\begin{cases} p_5-(p_1p_3+p_2p_4)\cos\theta(t)-(p_2p_3-p_1p_4)\sin\theta(t)-p_1r_x(t)-p_2r_y(t)=0 \\ p_1^2+p_2^2=1 \end{cases} \quad (1)$$

Contact State 2

$$\begin{cases} -q_5+(q_1q_3+q_2q_4)\cos\theta(t)-(q_2q_3-q_1q_4)\sin\theta(t)- \\ (q_1\cos\theta(t)-q_2\sin\theta(t))r_x(t)-(q_1\sin\theta(t)+q_2\cos\theta(t))r_y(t)=0 \\ q_1^2+q_2^2=1 \end{cases} \quad (2)$$

In these equations, the sensor path $S(t)$, $t\in\{t_0,t_1,\cdots,t_n\}$, consists of a finite set of locations and orientations of the robot gripper (i.e., $S(t)=\{r_x(t),r_y(t),\theta(t)\}$). Here, the minimum value of n is determined by the number of unknown parameters and the nature of the constraint equations. If additional sensors were available (e.g., force, torque), then additional constraint equations could be written. The sets $p=\{p_1,\ldots,p_5\}$ and $q=\{q_1,\ldots,q_5\}$ comprise the unknown time-independent parameters in the constraint equations.

For these examples, contact state distinguishability and identifiability can be summarized as follows:

- *Contact states 1 and 2 are globally distinguishable if, given almost any sensor path $S(t)$, there is no solution for the parameter set $\{p,q\}$ such that that (1) and (2) are satisfied simultaneously.*
- *Contact state 1 (resp. state 2) is globally identifiable if, given almost any sensor path $S(t)$, there exists a unique solution set p (resp. q) that satisfies equation (1) (resp. equation (2)). If there are a finite number of solutions then contact state 1 (resp. contact state 2) is locally identifiable.*

A. Distinguishability

In the robotics literature, distinguishability tests are local results which are employed during contact state estimation. Contact state estimation is a dual estimation problem in which contact states as well as properties parameterizing the states are estimated. A unified solution to these problems is provided using multiple model estimation [6]. With this technique, the parameters of all contact states are first estimated simultaneously in a moving data window; then the contact state is estimated using a distinguishability test based on the residuals of the parameter estimation. Additional examples include [14] and [9] where properties are estimated using a Kalman filter, and contact states are estimated using distinguishability tests based on the innovations of the filter. These tests include hypothesis testing [9], hidden Markov models [6], SNIS tests [14] and ratio of residual sum of squares [2]. The reported distinguishability tests are all local and numerical techniques.

B. Identifiability

Identifiability has been investigated in the robot calibration literature. For example, confidence intervals on the identified parameters [12] and condition number of the Jacobian matrix [19] can be used as local tests for identifiability. In particular a model is locally unidentifiable if the parameter Jacobian is singular (i.e, high condition number), and a model is locally identifiable if the Jacobian is far from singularity (i.e., condition number less than 100 [19]).

The only global result related to contact identifiability known to the authors is the notion of C-space equivalence defined by Eberman in [10]. To illustrate this approach, (1) is recast in an input-output form in (3), with $\{r_x(t),\theta(t)\}$ as the inputs and $r_y(t)$ as the output. Note that other input-output representations are possible.

$$\begin{pmatrix} r_y(t) \\ 1 \end{pmatrix} = \begin{bmatrix} -\left(\frac{p_1p_3+p_2p_4}{p_2}\right) & -\left(\frac{p_2p_3-p_1p_4}{p_2}\right) & -\left(\frac{p_1}{p_2}\right) & \left(\frac{p_5}{p_2}\right) \\ 0 & 0 & 0 & p_1^2+p_2^2 \end{bmatrix} \begin{bmatrix} \cos\theta(t) \\ \sin\theta(t) \\ r_x(t) \\ 1 \end{bmatrix} \quad (3)$$

The model is globally identifiable if the inverse mapping from the parameter matrix $F(p_i)\in\mathbb{R}^{2\times4}$ to $p_i\in\mathbb{R}^5$ is unique; it is locally identifiable if there are a finite number of solutions and it is unidentifiable if there are an infinite number of solutions. For this example, it can be easily shown that if $p_2\neq0$ and if the sign of the normal is known (i.e., inward or outward), then the mapping is unique resulting in global identifiability of contact state 1.

This technique can be applied only if the constraint equations can be written as affine functions of the inputs. Contact 2 in (2) provides a counterexample which cannot be written in this form.

C. State Space Model Distinguishability and Identifiability

State space models have been actively investigated for global distinguishability [21], [18], as well as local and global identifiability [15], [3], [13]. These tests have been used in applications related to control [15], biology [3], and chemistry [21].

In a series of papers published in the mid-eighties [13], [22], [21], Walter and Lecourtier provided a uniform way for testing the distinguishability and identifiability of state-space models. In these papers, the following model is used to represent the input-output relationship of experimental data.

$$M_i(p):\begin{cases} \dot{x}(t)=f\left(x(t),u(t),p,t\right) \\ y_i(t,p)=g\left(x(t),p,t\right) \end{cases}, x(0)=x_0,\ u(0)=u_0 \quad (4)$$

Here, x is the state vector, p is a set of unknown time-independent parameters, u is the input vector, and y is the output vector.

Two state space models $M_1(p)$ and $M_2(q)$ are globally distinguishable if (i) for almost any q there is no p such that $y_1(t,p) = y_2(t,q)$ and (ii) for almost any p there is no q such that $y_2(t,q) = y_1(t,p)$ for any input and time [23]. Similarly, a state-space model $M(p)$ is globally (locally) identifiable if for almost any q there is only one (a finite number of) p such that $y(t,p) = y(t,q)$ for any input and time [23].

There exist a variety of techniques in the literature for solving state space model distinguishability and identifiability. For linear models, these methods include equating transfer function coefficients [22] and similarity transformations [20]. For nonlinear models, techniques include linearization [11], Taylor series expansions [17] and generating series [23].

This paper focuses on the Taylor series expansion technique. In this approach, the Taylor series of the output vector at $t = 0^+$ can be written as a succession of time derivatives evaluated at time $t = 0^+$ as described in (5)-(6). It is assumed that the functions f and g and the vectors x and u are infinitely differentiable with respect to time.

$$y(t,p) = \sum_{k=0}^{n} a_k(p, x_0, u_0)\frac{t^k}{k!} \tag{5}$$

$$a_k(p, x_0, u_0) = \left.\frac{d^k y(t,p)}{dt^k}\right|_{t=0^+} \tag{6}$$

Note that each coefficient of the Taylor series (6) is a function of the unknown time-independent parameters $\{p, x_0, u_0\}$. Because of the uniqueness of the series coefficients, the input-output behavior of a state-space model can be written as a unique set of algebraic equations in which each equation corresponds to a coefficient of the series.

In order to test for global distinguishability and identifiability, the set of algebraic equations must be solved for all possible sets of parameters (i.e., find all the $p^* = \{p, x_0, u_0\}$ given the q^* such that $y_1(t,p^*) = y_2(t,q^*)$ or $y(t,p^*) = y(t,q^*)$).

These sets of equations can be difficult to solve by hand, however, tools from commutative algebra can be used to simplify the equations [18]. In this paper, algebraic sets of equations are transformed to Gröbner bases. Gröbner bases possess the same roots than the original system, but are often easier to solve [1].

III. Taylor Series Testing of Distinguishability and Identifiability for Contact States

As given in (7), the constraint equations representing a contact state contact equations are permitted to be nonlinear in the parameters p_i as well as the sensor variables $s_j(t)$. It is assumed here that the objects in contact are rigid and so their intrinsic properties p_i are time-independent.

$$\begin{cases} F\left(p_i, s_j(t)\right) = 0 \quad i = 1\ldots n, j = 1\ldots m \\ H(p_i) = 0 \end{cases} \tag{7}$$

$F(\cdot)$ includes all the sensor-dependent constraint equations while $H(\cdot)$ models any additional equality constraint on the parameters (e.g., $H(p) = p_1^2 + p_2^2 - 1$ in (1)).

Using the implicit function theorem, $F(\cdot)$ in (7) can be transformed into a set of input-output equations. Equation (8) presents the scalar result of this transformation; however multi-input, multi-output systems can also be considered.

$$\begin{cases} y(t) = G\left(p_i, u_j(t)\right) \quad i = 1\ldots n, j = 1\ldots m-1 \\ H(p_i) = 0 \\ \dfrac{\partial F}{\partial y} \neq 0 \end{cases} \tag{8}$$

The selection of the inputs and outputs from the sensor variables is not unique; however, the same choice must be made for all models when testing distinguishability. In this paper, the selection resulting in the simplest input-output models is chosen.

Adapting the results of [17], contact state constraint equations can be represented by Taylor series expansions. Since the equations are algebraic, the expansion is performed with respect to the inputs. In this paper, we consider only two-input, single-output systems, which includes the examples of (1) and (2). The approach, however, can be applied to any algebraic system. Assuming that the function $y(p,u)$ is analytic, a Taylor series expansion of order m can be written as:

$$y(p, u_1, u_2) = \sum_{n=0}^{m}\sum_{k=0}^{n} a_{nk}(p, u_{10}, u_{20})\frac{\Delta u_1^{n-k}\Delta u_2^{k}}{n!} \tag{9}$$

$$a_{nk}(p, u_{10}, u_{20}) = \left.\frac{\partial^n y(p, u_1, u_2)}{\partial u_1^{n-k}\partial u_2^{k}}\right|_{\substack{u_1=u_{10}\\ u_2=u_{20}}} \tag{10}$$

Since the function $y(p,u)$ is assumed to be infinitely differentiable with respect to its inputs, its mixed derivatives are equal. The number of coefficients n_c of the series is given by (11) where n_o is the number of outputs, n_i is the number of inputs, and m is the order of the expansion.

$$n_c(n_o, n_i, m) = n_o\frac{\prod_{k=1}^{n_i}(m+k)}{n_i!} \tag{11}$$

A. Distinguishability

Based on the uniqueness of the Taylor series expansion, two contact state models will possess the same input-output behavior if and only if all the coefficients of their expansions are equal, as given below.

$$y_1(u_1,u_2,p)=y_2(u_1,u_2,q)\Leftrightarrow\begin{cases}a_{00}(p,u_{10},u_{20})=b_{00}(q,u_{10},u_{20})\\ a_{10}(p,u_{10},u_{20})=b_{10}(q,u_{10},u_{20})\\ \vdots\\ a_{mm}(p,u_{10},u_{20})=b_{mm}(q,u_{10},u_{20})\\ H_1(p)=H_2(q)\end{cases}\qquad(12)$$

Thus, for two models M_1 and M_2 to be distinguishable, their outputs $y_1(u_1,u_2,p)$ and $y_2(u_1,u_2,q)$ must differ in at least one term of (10) for all choices of parameters. This test can be stated precisely as follows.

Definition 1: *Two contact state models M_1 and M_2, described by their output behavior $y_1(\cdot)$ and $y_2(\cdot)$, are globally distinguishable if and only if for almost any set of inputs $u(t)$, (i) given almost any set of parameters p, there is no solution to (12) for q, and (ii) given almost any set of parameters q, there is no solution to (12) for p.*

B. Identifiability

The identifiability of a contact model M described by its output behavior, $y(\cdot)$, can be tested by considering how many sets of parameters yield the same coefficients in (10). This test can be performed by counting the number of solutions for p, given q, in

$$y(u_1,u_2,p)=y(u_1,u_2,q)\Leftrightarrow\begin{cases}a_{00}(p,u_{10},u_{20})=a_{00}(q,u_{10},u_{20})\\ a_{10}(p,u_{10},u_{20})=a_{10}(q,u_{10},u_{20})\\ \vdots\\ a_{mm}(p,u_{10},u_{20})=a_{mm}(q,u_{10},u_{20})\\ H(p)=H(q)\end{cases}\qquad(13)$$

This test can be stated formally as follows.

Definition 2: *A contact state model M is globally identifiable if and only if, given any set of parameters q and almost any set of inputs $u(t)$, there is a unique solution to (13), which is $p=q$. If a finite number of solutions for p exist then M is locally identifiable. M is unidentifiable if an infinite number of solutions exist.*

C. Application of the Tests

Since (12) and (13) involve n unknown parameters, at least n independent algebraic equations are needed for their solution. Equation (8) can provide $n_c+\beta$ equations where n_c is the number of coefficients in the expansion and β is the dimension of $H(\cdot)$. The resulting lower bound on the number of Taylor coefficients is:

$$n_c \ge n-\beta \qquad (14)$$

This is a lower bound since there is no guaranty that the algebraic equations from the Taylor series are independent. This bound can be related to the order of the expansion by (11).

To solve for the parameters, the algebraic equations of (12) or (13) are transformed to a Gröbner basis using a computer algebra package (e.g., Mathematica). The number of solutions to the resulting equations is then determined to apply the distinguishability or identifiability test.

Since the Taylor series is developed around nominal input values, the approach appears to be local. It is important to note, however, that the solutions for the parameters are obtained without substituting numerical values for the inputs. Thus, the results are truly global.

IV. Examples

Three examples are presented using the contact states of Figure 1. The first two are identifiability tests of the individual contact states. The third tests the distinguishability of the two models.

A. Identifiability of Contact State 1

Since the contact state described by (1) can be expressed in input-output form as an affine function of the inputs, it was possible to use Eberman's method to test identifiability in section II.B.

To compare this result with the Taylor series approach, (1) is rewritten in the form of (8). To match the input-output choice of (3), $r_y(t)$ is taken as the output with $r_x(t)$ and $\theta(t)$ as the inputs. While this restricts p_2 to be nonzero, choosing $r_x(t)$ as the output would instead force p_1 to be nonzero.

$$\begin{cases}y(t)=\dfrac{p_5-(p_1p_3+p_2p_4)\cos u_1(t)-(p_2p_3-p_1p_4)\sin u_1(t)-p_1u_2(t)}{p_2}\\ p_1^2+p_2^2-1=0,\ p_2\neq 0\end{cases}\qquad(15)$$

Since there are five parameters, at least five equations are needed. The last equation of (15) provides one and so at least four series coefficients are needed. Through second order, these are:

$$\begin{cases}a_{00}=\dfrac{p_5-(p_1p_3+p_2p_4)\cos u_{10}-(p_2p_3-p_1p_4)\sin u_{10}-p_1u_{20}}{p_2}\\ a_{10}=\dfrac{-(p_2p_3-p_1p_4)\cos u_{10}+(p_1p_3+p_2p_4)\sin u_{10}}{p_2}\\ a_{11}=-\dfrac{p_1}{p_2}\\ a_{20}=\dfrac{(p_1p_3+p_2p_4)\cos u_{10}+(p_2p_3-p_1p_4)\sin u_{10}}{p_2}\\ a_{21}=a_{22}=0\end{cases}\qquad(16)$$

Equation (16) provides four independent equations. Due to the cyclic nature of the derivative of sine and cosine, additional terms in the expansion do not generate independent equations. The Taylor series coefficients of (16) combined with the last equation of (15) are used to form equations in the form of (13). These algebraic equations are transformed into a Gröbner basis as shown in (17).

$$\begin{cases} -p_1^2 + q_1^2 = 0 \\ p_2 q_1 - p_1 q_2 = 0 \\ 1 - p_1^2 - p_2^2 = 0 \\ p_2\left((p_3 - q_3)\cos u_{10} + (-p_4 + q_4)\sin u_{10}\right) = 0 \\ p_2\left((-p_3 + q_3)\sin u_{10} + (-p_4 + q_4)\cos u_{10}\right) = 0 \\ p_2(p_4 - q_4) = 0 \\ -p_5 q_2 + p_2 q_5 = 0 \end{cases} \tag{17}$$

Since $p_2 \neq 0$, it is easy to show that this system admits two solutions

$$\begin{array}{ll} p_1 = q_1 & p_1 = -q_1 \\ p_2 = q_2 & p_2 = -q_2 \\ p_3 = q_3 \quad , & p_3 = q_3 \\ p_4 = q_4 & p_4 = q_4 \\ p_5 = q_5 & p_5 = -q_5 \end{array} \tag{18}$$

As was the case in section II.B, the solution can be uniquely determined if the sign of the contact normal (i.e., p_1, q_1) is known. By Definition 2, contact state 1 is globally identifiable if $\mathrm{sgn}(p_1)$ is known and locally identifiable otherwise. Note that while the choice of output restricted p_2 to be nonzero, by choosing $r_x(t)$ as the output, $p_1 \neq 0$ instead. Repeating the solution process for this case yields the same result.

B. *Identifiability of Contact State 2*

From (2) it can be observed that this model cannot be made affine in the inputs regardless of the choice of output. This necessitates the use of the Taylor series method and so (2) is rewritten in the form of (8). As in the previous example, $r_y(t)$ is selected as the output with $r_x(t)$ and $\theta(t)$ as the inputs:

$$\begin{cases} y(t) = \dfrac{\begin{pmatrix} -q_5 + (q_1 q_3 + q_2 q_4)\cos u_1(t) - (q_2 q_3 - q_1 q_4)\sin u_1(t) \\ -(q_1 \cos u_1(t) - q_2 \sin u_1(t))u_2(t) \end{pmatrix}}{(q_1 \sin u_1(t) + q_2 \cos u_1(t))} \\ q_1^2 + q_2^2 - 1 = 0,\ q_1 \sin u_1(t) + q_2 \cos u_1(t) \neq 0 \end{cases} \tag{19}$$

Series coefficients are computed to second order and are given in (20). Using (13), these coefficients provide five equations which are combined with the last equation of (19).

$$\begin{cases} b_{00} = \dfrac{\begin{pmatrix} -q_5 + (q_1 q_3 + q_2 q_4)\cos u_{10} - (q_2 q_3 - q_1 q_4)\sin u_{10} - \\ (q_1 \cos u_{10} - q_2 \sin u_{10})u_{20} \end{pmatrix}}{q_1 \sin u_{10} + q_2 \cos u_{10}} \\ b_{10} = \dfrac{q_1 q_5 \cos u_{10} - q_2 q_5 \sin u_{10} - (q_1^2 + q_2^2)(q_3 - u_{20})}{(q_2 \cos u_{10} + q_1 \sin u_{10})^2} \\ b_{11} = \dfrac{-q_1 \cos u_{10} + q_2 \sin u_{10}}{q_2 \cos u_{10} + q_1 \sin u_{10}} \\ b_{20} = \dfrac{\begin{pmatrix} 3q_2^2 q_5 + (q_1^2 - q_2^2) q_5 \cos(2u_{10}) + 4q_1^2 q_2 q_3 \sin u_{10} \\ +4q_2^3 q_3 \sin u_{10} - 2q_1 q_2 q_5 \sin(2u_{10}) - 4q_1^2 q_2 \sin u_{10} u_{20} - \\ 4q_1(q_1^2 + q_2^2)\cos u_{10}(q_3 - u_{20}) - 4q_2^3 \sin u_{10} u_{20} + 3q_1^2 q_5 \end{pmatrix}}{-2(q_2 \cos u_{10} + q_1 \sin u_{10})^3} \\ b_{21} = \dfrac{q_1^2 + q_2^2}{(q_2 \cos u_{10} + q_1 \sin u_{10})^2} \\ b_{22} = 0 \end{cases} \tag{20}$$

The system of algebraic equations is transformed to a Gröbner basis. For lack of space, the resulting basis is not written.
As with the previous example, contact state 2 is globally identifiable if the sign of the contact normal is known and locally identifiable otherwise.

C. *Distinguishability of Contact States 1 and 2*

To test distinguishability of the models, Definition 1 is applied to the equations formed by combining (16) and (20) using (12) together with the final equations in (15) and (19). It can be directly observed that the equation $a_{21} = b_{21}$ cannot be satisfied since $q_1^2 + q_2^2 = 1$ and $-1 \leq q_1 \sin u_1(t) + q_2 \cos u_1(t) \leq 1$.

$$a_{21} = b_{21} \Leftrightarrow \frac{q_1^2 + q_2^2}{(q_2 \cos u_{10} + q_1 \sin u_{10})^2} = 0 \tag{21}$$

By Definition 1, since the equations have no solution regardless of whether p or q is given, the two contact state models are globally distinguishable.

V. CONCLUSIONS

This paper has provided the first method for testing the global distinguishability and identifiability of contact state models. Just as contact state estimation is a dual problem involving the estimation of both state parameters as well as contact states, the Taylor series method provides a unified approach to testing the capability to estimate both the parameters and the states.

The approach can be applied to any smooth nonlinear constraint equations. Possible limitations of the method include no known bound on how many coefficients are needed as well as the complexity of solving the resulting algebraic equations.

In contrast to the local numerical methods currently applied in parameter and contact state estimation, however, no sensor data is needed for the tests. Since the equations involving the Taylor series coefficients are evaluated symbolically, the results are *not* local.

Furthermore, the dependence of identifiability and distinguishability on those sensors selected as inputs can be assessed. Future work will consider the effect of input excitation on identifiability and distinguishability.

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