

# Electromechanical emulation of active vibratory systems

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**Abstract:** The design of a simple electromechanical system which is dynamically equivalent to an active vibratory system is studied in this paper. A two-step process is presented in which a passive vibratory system is first obtained, which is then modified in the second step to achieve active equivalence. Implementation of the active emulation step is achieved by closed-loop control of electromechanical shakers attached to the passive system and driven so as to generate the appropriate vibrations at the mounting locations of the active vibratory system. Experimental examples are used to demonstrate the effectiveness of this design process.

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## 1. Introduction

The electromechanical emulation problem concerns the design and fabrication of dynamically equivalent models for complicated structures. This problem arises in the scaled vibration testing of novel superstructures or foundations for ships, submarines, and space structures. Equipment and machinery attached to the superstructure can both absorb and generate vibrational energy and so testing must also include its dynamics. In such testing, scaled models of the superstructure are often easy to construct. The scaled modeling of electronic equipment and power plant machinery is not, however, as straightforward. In these situations, an economical alternative to detailed scaling is the use of emulators—simple electromechanical systems which are dynamically equivalent only at the mounting locations where the equipment is attached to the superstructure.

The electromechanical filter design problem is similar except that, in this case, one starts with an actual machine instead of a desired frequency response function. Experiment or finite element simulation is used to extract the mounting location dynamic response, usually in the form of accelerance. Subsequently, mathematical models are fit to the data. These models are of a form that can be directly interpreted as an interconnection of easily realizable electromechanical elements.

A variety of techniques have been developed for passive emulation which specifically address the identification of, or conversion to, a model in realizable form. See, for example, Refs. 1–4.

Active emulation, which is the topic of this paper, involves reproducing the internally-generated vibrations which are transmitted to the superstructure. These vibrations are caused by such elements as unbalanced rotors and are mainly determined by the amplitudes and directions of the internal exciting forces, the transmission paths from those forces to the mounting locations, and the dynamic interaction between active machinery and its mounting structures.

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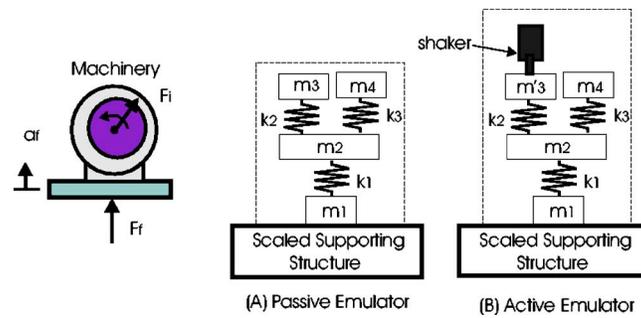


Fig. 1. (Color online) Two-step machinery emulator design procedure.

Active emulation has apparently not been addressed in the literature. The approach presented here is to modify a passive emulator to achieve active emulation. The method is described in the next section and is followed by an experimental validation.

## 2. Solution approach

The active emulation problem is solved by a two-step procedure as illustrated in Fig. 1. In the first step, a passive mechanical emulator is designed to match the passive dynamics of the machinery. The passive emulator must possess the rigid body modes and passive frequency responses of the original machinery within the frequency range of interest. It is assumed that this model is obtained by one of the existing methods in the literature. In the second step, this passive emulator is modified by adding active elements, such as shakers, to produce the active vibrations at the foundation mounting locations.

This approach assumes linearity of the models, which is typically valid for above-mount machinery models, i.e., models which are defined to exclude elastomeric mounts. Modeling of elastomeric mounts is treated elsewhere, e.g., Ref. 5. Using superposition, the total acceleration,  $a_f$ , at the mounting locations due to forces from both the superstructure and from internal moving components is given by

$$a_f = a_f^p + a_f^a = A_{ff}f_f + A_{fi}f_i. \quad (1)$$

The dimension of the vector  $a_f$  is the product of the number of mounting locations and the degrees of freedom per mounting location. The passive and active accelerations are denoted  $a_f^p$  and  $a_f^a$ , respectively.  $A_{ff}$  is the drive-point accelerance matrix of the passive (nonoperating) machinery and  $f_f$  is the vector of forces exerted by the superstructure on the passive machinery. Experimentally,  $A_{ff}$  can be estimated by suspending the machinery and driving the mounting locations with shakers through impedance heads.  $A_{fi}$  is the transfer accelerance matrix between the active internal forces  $f_i$  and the mounting locations. While  $f_i$  cannot usually be measured,  $a_f^a$  can be measured, for example, by suspending the operating machinery and measuring the accelerations of the freely-suspended mounting locations.

The passive emulator is designed to reproduce  $A_{ff}$  at its mounting locations. The active accelerations,  $a_f^a$ , are reproduced by attaching shakers to the passive emulator and driving them appropriately. Usually, at least one shaker should be used to reproduce motion in each coordinate direction. By using two emulating shakers, rotational accelerations may be generated.

In the approach adopted here, emulating shakers are commanded to produce the force corresponding to the mounting location acceleration occurring when there is no external loading. As given in Eq. (1), this technique provides for total mounting location acceleration to be a linear combination of the effects of internal and external forcing. It is assumed that external excitation does not affect the equivalent internal forcing of operating machinery.

This approach to active emulator design involves three aspects: selection of the emulating shakers, selection of their mounting locations in the passive emulator, and design of a controller for the shakers. These are discussed below.

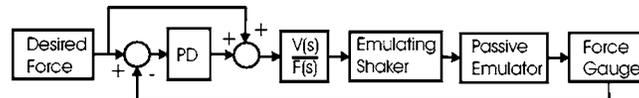


Fig. 2. Feedback control of the emulating shaker.

### 2.1 Selection of emulating shakers and their mounting locations

An emulating shaker must satisfy both passive and active matching requirements. Since the mass and rotational inertia of a shaker are appreciable, the mass element of the passive emulator to which the shaker is attached must be modified accordingly. For the single degree of freedom example in Fig. 1, the mass  $m'_3$  must satisfy

$$m'_3 = m_3 - m_s > 0, \quad (2)$$

where  $m_s$  is the emulating shaker mass. In general, approximating the nondriven shaker as a rigid mass, the combined shaker and emulator mass element should meet all passive rigid body matching requirements (see, e.g., Ref. 6).

For active emulation, a shaker must meet or exceed the force magnitude required within the frequency range of interest. The required magnitude is strongly influenced by the shaker's mounting location in the passive emulator. Consequently, while any location satisfying the passive matching requirements is possible, the emulator's base frame is often the best location since it typically maximizes the transfer acceleration to the emulator mounting locations permitting use of a smaller shaker.

Given a set of emulating shaker mounting locations, the forces which they should produce can be computed using  $A_{fi}^e(\omega)$ , the transfer accelerance matrix relating shaker forces to emulator mounting location accelerations. This accelerance matrix can be experimentally determined or theoretically obtained from the passive emulator model. The shaker forces  $F_i^e(\omega)$  are the solution of

$$a_f^a(\omega) = A_{fi}^e(\omega)F_i^e(\omega). \quad (3)$$

Considering linear coordinates (for simplicity of presentation), an upper bound on the number of emulating shakers is given by  $\dim[a_f^a(\omega)]$ , i.e., one for every coordinate direction at every mounting location. In this case,  $A_{fi}^e(\omega)$  is a square matrix and Eq. (3) is solved by matrix inversion over the frequency range of interest.

Since an emulator models a single machine or piece of equipment, in many practical situations, the actual number of emulating shakers needed is less than  $\dim[a_f^a(\omega)]$ . Mathematically, a surplus of shakers would appear in Eq. (3) as a loss of column rank in  $A_{fi}^e(\omega)$ . By examining  $A_{fi}^e(\omega)$ , one can deduce which emulating shakers can be omitted while still ensuring that  $a_f^a(\omega)$  lies in the column space of  $A_{fi}^e(\omega)$ . In these cases,  $\dim[F_i^e(\omega)] < \dim[a_f^a(\omega)]$  and a least squares solution of Eq. (3) can be employed.

### 2.2 Emulating shaker controller

For the shakers to produce the forces calculated from Eq. (3), they must be driven under closed-loop control. For example, the controller of Fig. 2 utilizes proportional and derivative (PD) feedback together with a feedforward term to obtain the voltage to be applied to the emulating shaker. The dynamic model,  $V(s)/F(s)$ , relating shaker voltage and force over the frequency range of interest, can be readily obtained from experiment using model estimation of sinusoidal sweep data.

In calculating the emulator forces  $F_i^e(t)$  from Eq. (3), if the acceleration is composed of several dominant harmonics, the force is likely to be dominated by these frequencies. In this

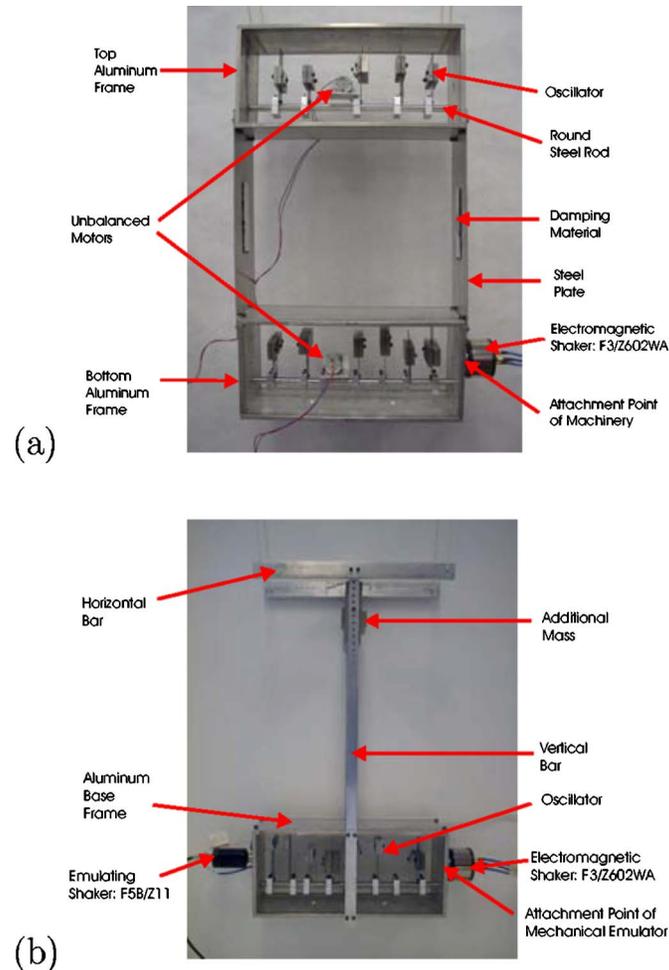


Fig. 3. (Color online) (a) Machinery test bed. (b) Emulator test bed. Both systems are suspended for testing.

case, the amplitude and phase angle of each frequency component in the forces can be evaluated discretely. Otherwise, the time-domain forces are obtained from the inverse Fourier transformation of  $F_i^e(\omega)$ .

### 3. Experimental example

To evaluate the proposed approach to active emulation, a machinery test bed was constructed as shown in Fig. 3(a). Use of this test bed instead of an actual machine provided the ability to vary the number, frequency, and damping of the passive modes as well as the internal forcing.

The depicted system is comprised of two aluminum frames connected by steel plates. A total of 11 oscillators are mounted on the top and bottom frames whose fixed-base frequencies can be tuned between 10 Hz and 80 Hz. The system also possesses a structural mode at about 25 Hz associated with bending of the steel plates. Polymeric damping materials are attached to both the oscillators and the frame to achieve various levels of damping. Two dc motors with eccentric masses fixed to their shafts are mounted as shown in the figure. These motors, when not rotating, contribute two modes between 60 Hz and 70 Hz. The height of the test bed is 56 cm, its width is 30 cm, and its depth is 10 cm. The total mass of the test bed is 6.12 kg.

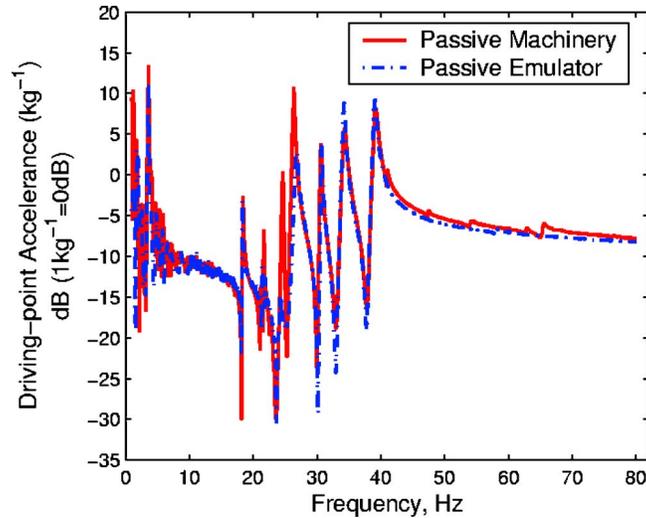


Fig. 4. (Color online) Horizontal mounting-point passive acceleration of machinery test bed and emulator.

A single superstructure mounting location is defined to be at the lower right corner where the testing shaker is shown attached. Three degrees of freedom are considered for this mounting location consisting of translations in the horizontal and vertical directions and rotations in the plane of the figure. As depicted, the test bed is suspended from four cords for testing purposes.

The passive emulator of Fig. 3(b) was designed using methods described in Ref. 6 to provide rigid-body matching in the vertical and rotational coordinate directions and dynamic matching between 10–80 Hz in the horizontal direction at the mounting location. A comparison of the horizontal direction acceleration is given in Fig. 4. Note that the two modes below 10 Hz are rigid-body suspension modes.

For active emulation, a single emulating shaker was needed. Its mounting location was selected on the base frame as shown in Fig. 3(b). The total mass of the emulating shaker and

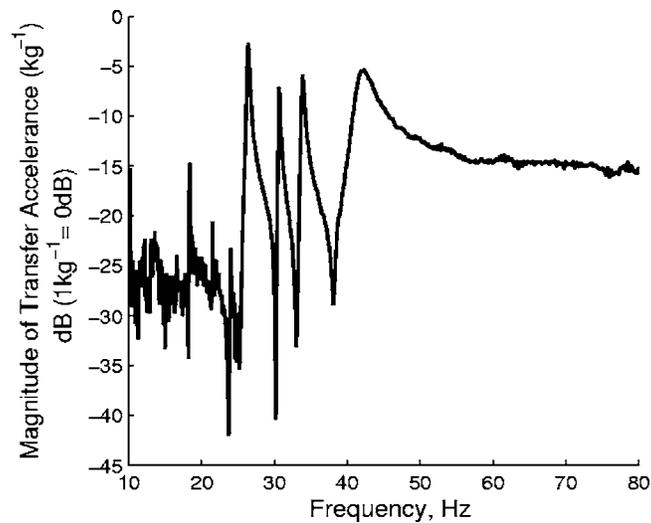


FIG. 5. Transfer acceleration,  $A_{f_i}^e(\omega)$ , of emulating shaker.

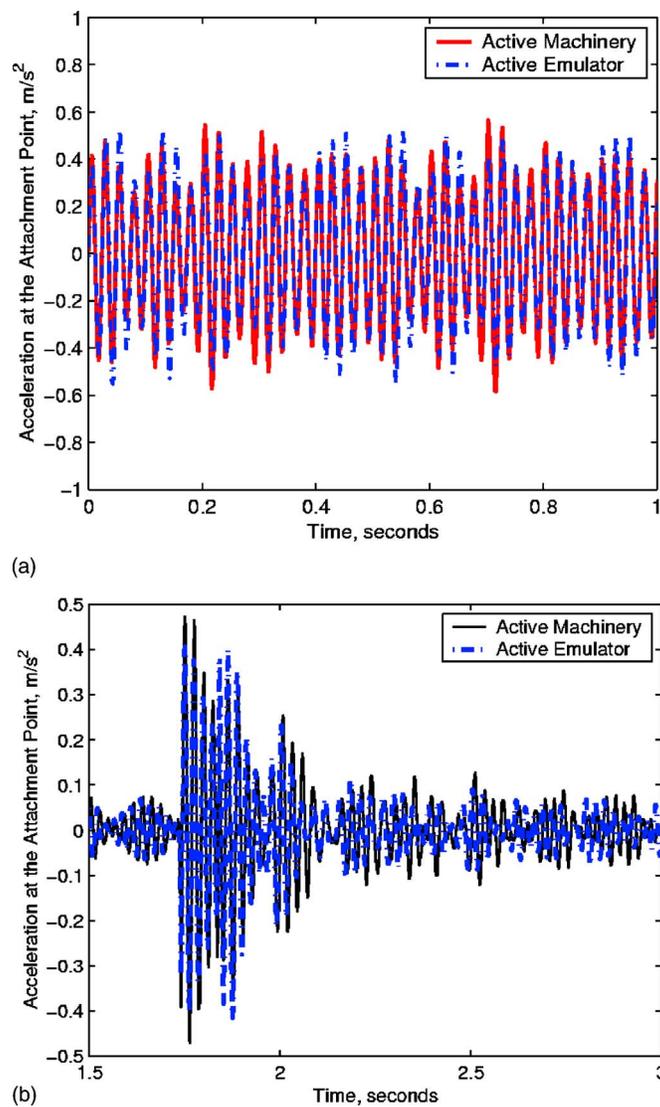


FIG. 6. (Color online) Response of active machinery and emulator. (a) Steady-state loading from superstructure. (b) Shock loading from superstructure.

impedance head is 0.170 kg. The transfer accelerance  $A_{rn}^e(\omega)$  from the emulating shaker location to the superstructure mounting location was measured between 10–80 Hz, as shown in Fig. 5.

To implement the controller of Fig. 2, the transfer function,  $V(s)/F(s)$ , was estimated from sinusoidal sweep data between 10–80 Hz to be

$$\frac{V(s)}{F(s)} = -\frac{2.007s^2 + 226.5s + 86670}{s^2 + 20s + 455}. \quad (4)$$

The feedback controller was implemented using a 16-bit AD/DA board and a personal computer.

Two experiments were carried out to verify active emulation. In both tests, the unbalanced motors of the machinery test bed were driven at frequencies of approximately 38 and

44 Hz. Given the discrete nature of the forcing, the emulating shaker forces were calculated using Eq. (3) and the data of Fig. 5 evaluated at the two discrete frequencies.

The first experiment simulates the case in which the superstructure applies periodic forcing to the system, such as might be produced by nearby machinery. The second experiment simulates the situation of the superstructure applying a shock loading to the system. These forcing signals were applied using a shaker located at the superstructure mounting location and the resulting force and acceleration were recorded using an impedance head.

### 3.1 Experiment 1: Periodic excitation

In this experiment, the horizontal forcing applied to the machinery and emulator mounting points was a weighted sum of five sinusoids of frequencies 20, 28, 32, 40, and 50 Hz. The mounting location response of the machinery test bed and emulator are depicted in Fig. 6(a). This is the total response due to forcing by the superstructure and by the internal active loading. The relative acceleration error magnitudes at the five superstructure forcing frequencies are all less than 10% and can be attributed to passive emulation error. These relatively large errors can be anticipated since the frequencies lie in a region of Fig. 4 in which large changes in magnitude occur for small changes in frequency. At the active vibration frequencies of 37 and 44 Hz, the relative emulation error magnitudes are a more modest 6.67% and 1.13%, respectively. Note from the transfer acceleration of Fig. 5, however, that the active emulation error can be similarly dependent on frequency.

### 3.2 Experiment 2: Impulsive excitation

The results of the impulsive loading experiment are shown in Fig. 6(b). The impulse is applied at about 1.75 s. The signal before this time corresponds to the emulating shaker matching the mounting location acceleration produced by the two unbalanced machinery motors. The relative magnitude error at the two motor frequencies is 3.23% and 0.95%. Just after application of the impulse, the response is dominated by the passive dynamics of the systems which are in good agreement. As the responses decay, however, small differences in the passive dynamics can be seen for times 2.25–3 s. Later times, not depicted, show recovery to the level of matching depicted prior to 1.75 s.

## 4. Conclusions

A two-step design process for active electromechanical emulation has been developed. In the first step, a passive mechanical emulator is designed to match the passive dynamic properties of a vibratory system. In the next step, this passive emulator is modified by the addition of active elements driven by a controller to reproduce the mounting location accelerations of the active vibratory system. While the approach has been demonstrated on a planar test bed, the extension of the method to higher dimensions is straightforward.

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